Math Virtual Learning

AP Statistics

Tests on a single mean

April 21, 2020

Lesson: April 21, 2020

Objective/Learning Target:
Students will be able to apply 1 sample t-test methods to determine significance.

Review #1

Which of the following are true statements:

1. The significance level of a test is the probability of a Type 2 error.
2. Given a particular alternative, the power of a test against that alternative is 1 minus the probability of the Type 2 error associated with that alternative.
3. If the significance level remains fixed, increasing the sample size will reduce the probability of a type 2 error.

Review #2

Given an experiment with H_0: p=0.25, H_a:p>0.25, and a possible correct value of 0.26, which of the following increase with an increase in the sample size n?

1. The probability of a Type 1 error
2. The probability of a type 2 error
3. The power of the test

Answers

1. 2 and 3 are true. The significance level of a test is the probability of a Type 1 error, not type 2.
2. Only 3 is correct. Increasing the sample size decreases the variation of both the null and alternative distributions. This results in less overlap of the two. The error results from the overlap, thus both types of error decrease with increased sample size. The power of the test does however increase.

Sampling distribution for sample means

This follows the same sampling distribution as that for the confidence interval. Recall…

Center: the mean of all sample means is equal to the population mean. Thus, sample means are unbiased estimators of population means and can be used for inference.

Spread: the standard deviation is $\frac{\sigma}{\sqrt{n}}$. However, we rarely know σ, so we use s_x instead. This is not a perfect estimate of σ, so we use the t-distribution to account for the error.

Shape: The central limit theorem applies here. We will address this on the next slide.

Recall the central limit theorem

When applying Central limit theorem. We know normal populations give normal sampling distributions. We also know that $n>30$, gives normal sampling distributions, but what about all the other cases? We will apply the following rule:

We can consider the data approximately normal if...

$n<15$, data roughly symmetric, single peak, no outliers

$n\geq 15$ no strong skew or outliers

$n\geq 30$

Assumptions for 1 sample t-test

In order to apply the 1 sample t-test, we need to meet three assumptions:

1. **Random:** The sample is from a random process, or the sample can fairly be considered representative
2. **Independent:** The process of sampling does not change the probability of the event happening, and if we are dealing with a finite population, that we sample less than 10% of the population.
3. **Normal:** We meet the one of the conditions of the central limit theorem. This might require making a graph.

These are the same assumptions as we made for the t-confidence interval! In fact the interval and test are equivalent.

Example

The maker of a hybrid car claims that the car will get 40 miles to the gallon (mpg). A random sample of 20 cars were tested to see if the mileage is not as high as claimed. Perform a significance test at the 5% level

37.2	38.4	38.9	40.0	42.3	39.7	37.8	39.2	40.8	39.9
37.9	39.2	40.4	40.0	38.7	39.8	42.8	41.1	38.9	39.5

State

We are going to test (at $\alpha=0.05$) whether the hybrid cars are really getting less than the 40 mpg claimed by the company. We will also estimate using a confidence interval.

μ: The true population mean miles per gallon of the hybrid cars.

H_0: $\mu = 40$ mpg

H_a: $\mu < 40$ mpg

Plan

1 sample t-test with $\alpha=0.05$

assumptions:

Random:The problem states the cars are from a random sample

Independent: It is safe to assume one cars mpg do not change the others. In addition, we can safely assume that 20 cars is less than 10% of the entire population.

Normal: This one gets its own slide… it is a bit more involved!

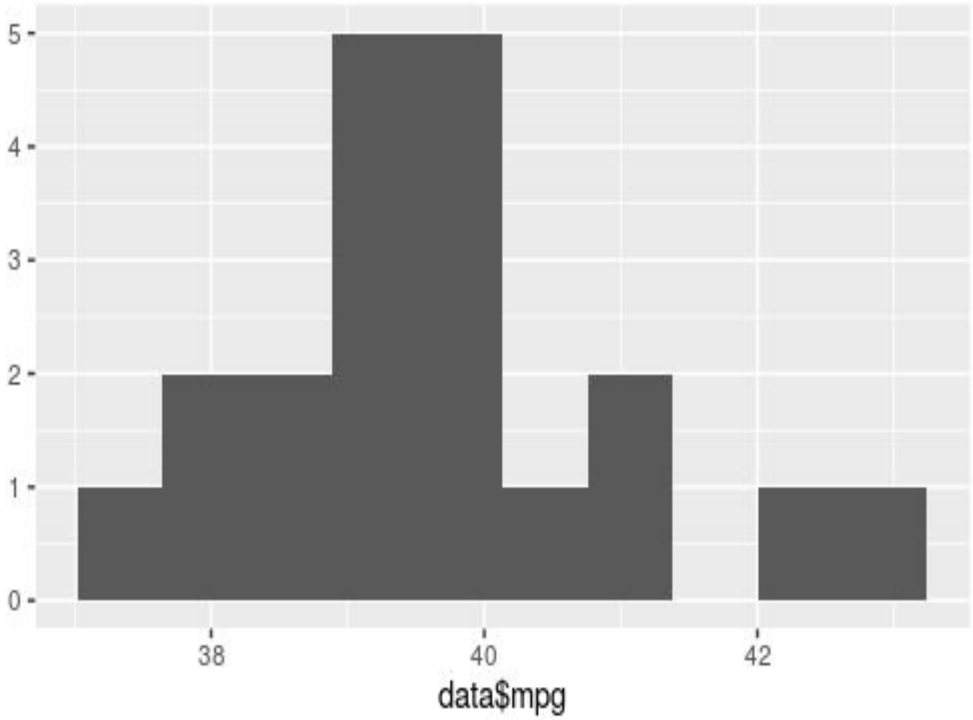
Proving normality

Recall… with a sample size of twenty, the central limit theorem only applies if we can show there are no outliers and no strong skewness. We need a plot to prove this. Histograms, box plots, and normal probability plots are all acceptable.

We will examine each of these plots, but only one is necessary.

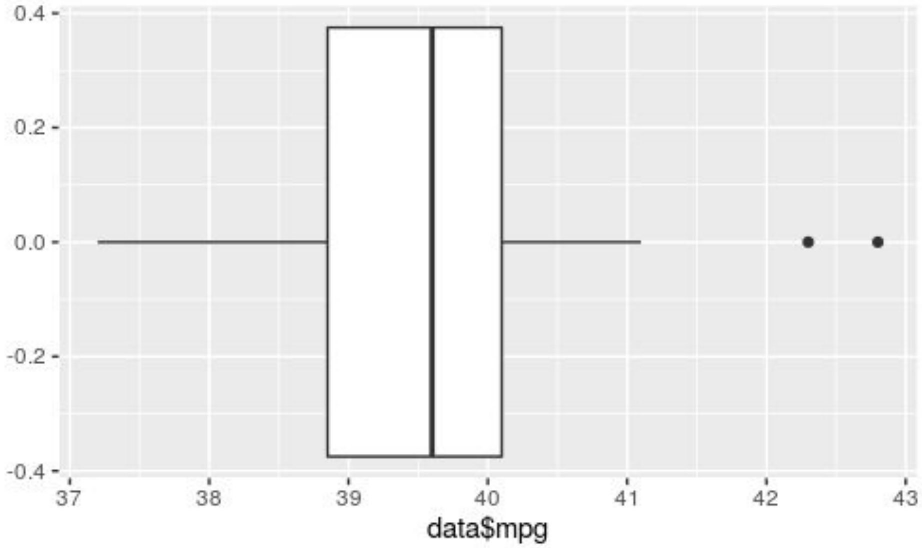
Proving normality

The histogram has no large outliers, but the values beyond 42 might be suspect. The distribution does seem to be very slightly skewed right. There does not appear to be a large deviation from what the central limit theorem allows. We will proceed with caution.



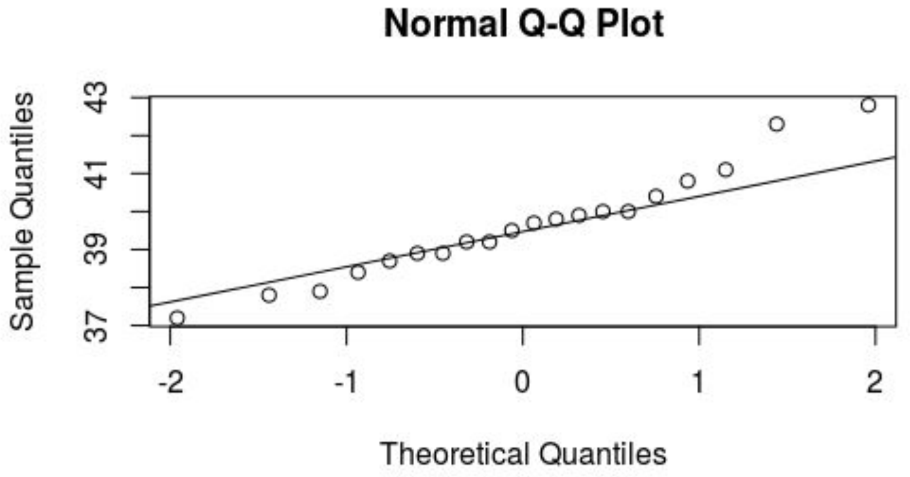
Proving normality

The boxplot does identify the points beyond 42 mpg as potential outliers. Other than those two points, the rest of the distribution does not appear to deviate from normality. The central limit theorem does not guarantee normality, but the t-distribution is robust. We will proceed with caution.



Proving normality

Examining the normal quantile plot, the data appears to follow a normal distribution for a majority of the distribution. The robustness of the t-distribution along with the central limit theorem should be able to handle the slight deviation from normality for the larger values. We will proceed with the test.



Normality

- All three explanations for normality are acceptable. All three of these plots can be created on the TI-84 calculator under the stat plot menu. It is your choice, but the plot must be created and used as part of the answer. Make sure to draw the plot.
- You might notice, we do not exactly meet the condition. However, t-distributions are robust. This means they can often still work when the assumptions are not met 100%. We must get comfortable deciding when close is close enough. In this case the two problem points are not a lot above the rest and might be the result of a slight skew in the data. We can handle a slight skew, so we decide to continue.

Do - by hand

Significance level = 0.05
Sample mean = 39.625
Sample standard deviation = 1.4097
n= 20
df=20-1=19
Critical value = -1.729 ← We look this up on the t-table for a single tail, then make it negative because it is a lower tail.

Put the original data in List 1 (STAT Enter) in your calculator and then do 1-variable stats (STAT → Calc Enter)

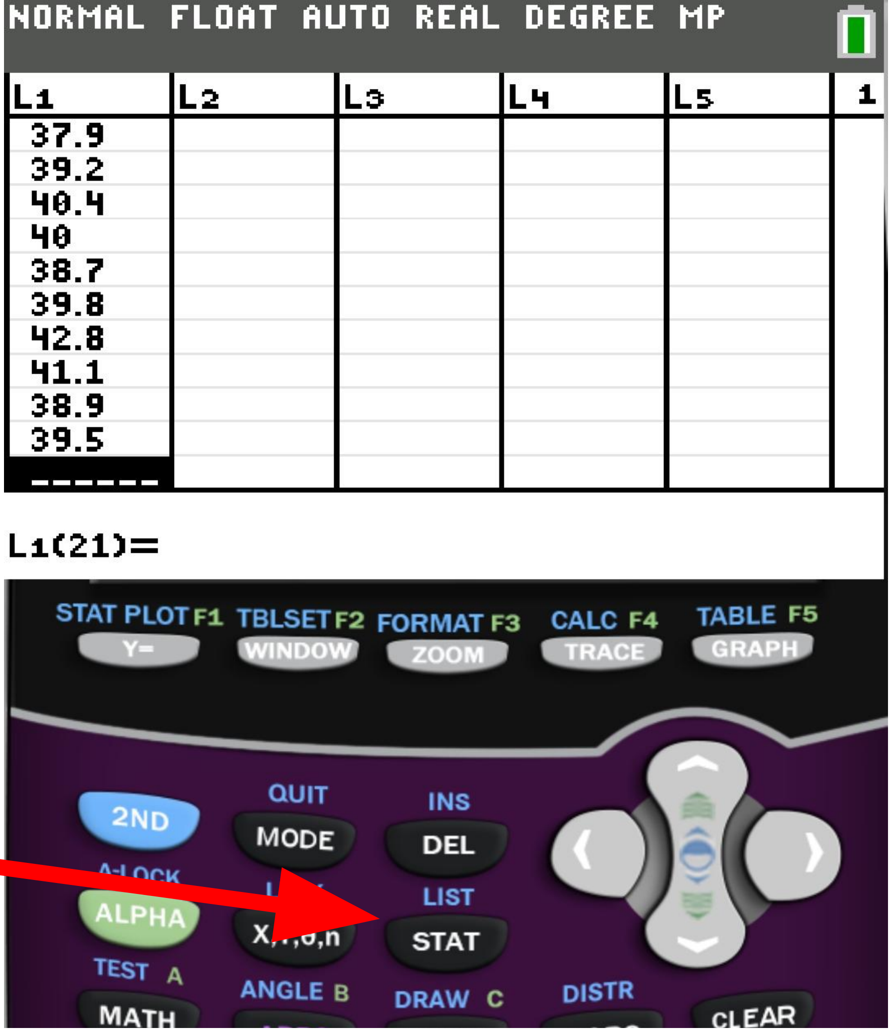
$$t^* = \frac{39.625-40}{\frac{1.4097}{\sqrt{20}}} = -1.19$$

Since the t* value is not beyond the critical value we do not have evidence to reject the null.

Do - by calculator

1. Enter the data in the list. Press stat → edit, then enter data into list 1.

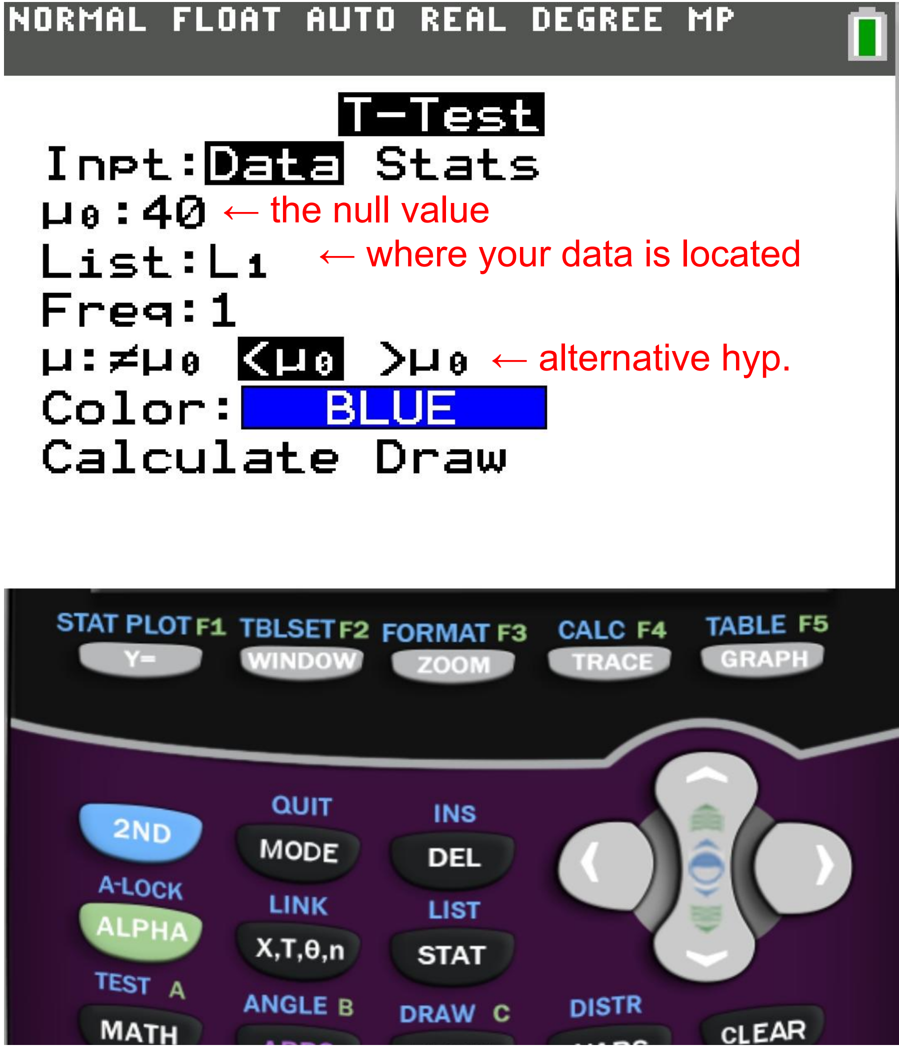
Here is the stat button if you forgot



Do - by calculator

Press stat → scroll right to Test → use the T-test function.

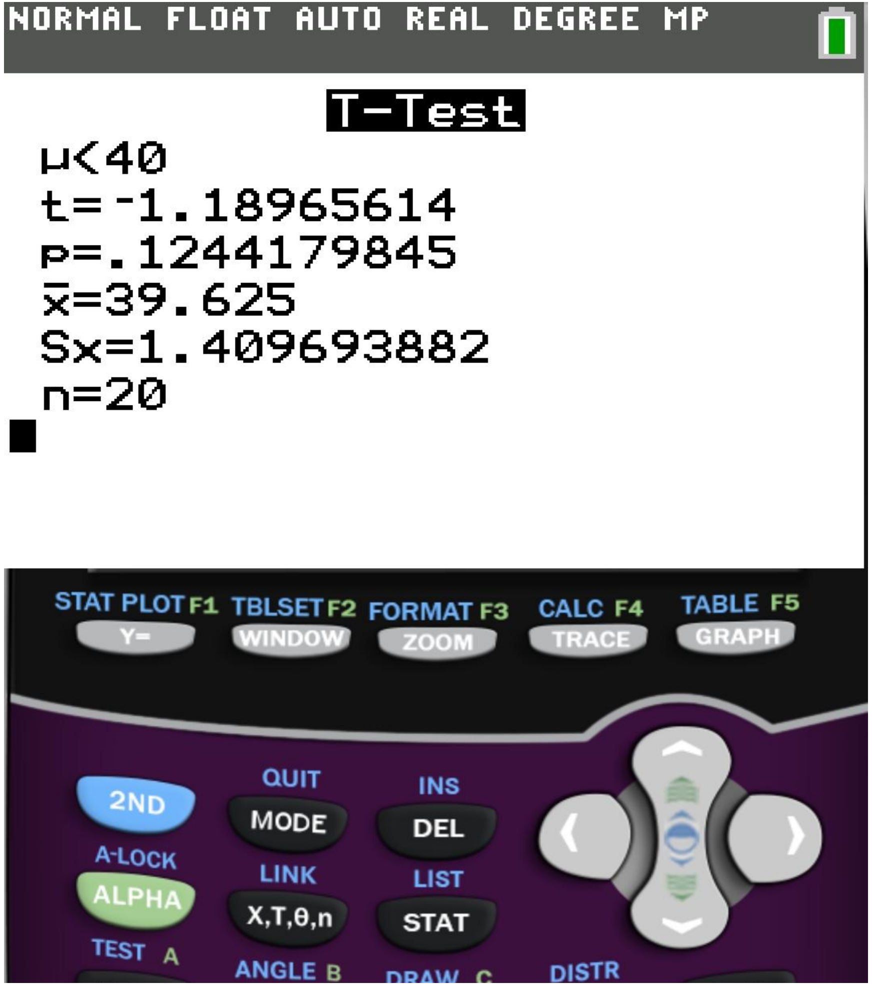
We have the option to use data or stats. Here we will use data as we have it in the calculator. Enter in the rest of the info indicated to the right, and press calculate.



Do - by calculator

Write down that you used the t-test function, and then copy ALL the results.

We notice that we have the same t* value of about -1.19. We also notice that p=0.1244… meaning that we are likely to get this value about 12.44% of the time given the null. We again would fail to reject.



Conclude

We do not have evidence to reject the null hypothesis that the true mile per gallon is less than 40 mpg, with a significance level of 0.05. We do not have evidence to conclude the true mpg is anything other than what the company claims.

Extra practice

Reading: pg 565-586

HW: 57–60, 71, 73, 75, 77, 89, 94–97, 99–104